Algebraic Properties [Axioms]

2009 *Mathematics Standards of Learning*

The algebraic properties listed apply given a, b, and c are real numbers. This is not an exhaustive list of algebraic properties.

Field Properties

Property	Addition	Multiplication
Associative	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
Commutative	$a + b = b + a$	$ab = ba$
Identity	$a + 0 = a = 0 + a$	$a \cdot 1 = a = 1 \cdot a$
Inverse	$a + (-a) = 0 = (-a) + a$	$a \cdot \frac{1}{a} = 1 = \frac{1}{a} \cdot a$, if $a \neq 0$
Distributive	$a(b + c) = ab + ac$ and $ab + ac = a(b + c)$	

Properties of Equality and Inequality

Property	Equality	Inequality
Multiplicative Property of Zero	$a \cdot 0 = 0 = 0 \cdot a$	
Zero Product	If $ab = 0$, then $a = 0$ or $b = 0$.	
Reflexive	$a = a$	
Symmetric	If $a = b$, then $b = a$.	
Transitive	If $a = b$ and $b = c$, then $a = c$.	If a > b and b > c, then a > c. If a < b and b < c, then a < c.
Addition	If $a = b$, then $a + c = b + c$.	If $a < b$, then $a + c < b + c$. If $a > b$, then $a + c > b + c$.
Subtraction	If $a = b$, then $a - c = b - c$.	If $a < b$, then $a - c < b - c$. If $a > b$, then $a - c > b - c$.
Multiplication	If $a = b$, then $ac = bc$.	If $a < b$ and $c > 0$, then $ac < bc$. If $a < b$ and $c < 0$, then $ac > bc$. If $a > b$ and $c > 0$, then $ac > bc$. If $a > b$ and $c < 0$, then $ac < bc$.
Division	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.	If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$. If $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$. If $a > b$ and $c > 0$, then $\frac{a}{c} > \frac{b}{c}$. If $a > b$ and $c < 0$, then $\frac{a}{c} < \frac{b}{c}$.
Substitution	If $a = b$, then b can be substituted for a in any equation or inequality.	

Vertical Articulation of Algebraic Properties

2009 *Mathematics Standards of Learning*	
Grade 3	3.20 The student will a) investigate the identity and commutative properties for addition and multiplication; and b) identify examples of the identity and commutative properties for addition and multiplication.
Grade 4	4.16 The student will b) investigate and describe the associative property for addition and multiplication.
Grade 5	5.19 The student will investigate and recognize the distributive property of multiplication over addition.
Grade 6	6.19 The student will investigate and recognize a) the identity properties for addition and multiplication; b) the multiplicative property of zero; and c) the inverse property for multiplication.
Grade 7	7.16 The student will apply the following properties of operations with real numbers: a) the commutative and associative properties for addition and multiplication; b) the distributive property; c) the additive and multiplicative identity properties; d) the additive and multiplicative inverse properties; and e) the multiplicative property of zero.
Grade 8	8.15 The student will c) identify properties of operations used to solve an equation.
Algebra I	A.4 The student will solve multistep linear and quadratic equations in two variables, including b) justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets. A.5 The student will solve multistep linear inequalities in two variables, including b) justifying steps used in simplifying expressions and solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets.
Algebra II	AII.3 The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of *i*, and identify field properties that are valid for the complex numbers.